# Voltage Deformable State Parallel Arrangement of Cylindrical Pipe with a Liquid under Harmonic Loads 

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Abstract:The paper deals with the stress-deformed state parallel cylindrical tubes with liquid. The problem is solved in the orientation of the cylindrical coordinate system under the influence of harmonic waves. An analytical solution for special Bessel functions and Hankel and numerical results. Parametric analysis of dynamic stress factor.

Keywords: a cylindrical tube, the liquid harmonic wave, bi cylindrical coordinate system, special features.
Some basic ratio of elasticity.
It is known that in a static elastic Lame theory equation in vector form is [1,2,3]:

$$
\begin{equation*}
(\lambda+2 \mu) \operatorname{grad} \operatorname{div} \overrightarrow{\mathrm{u}}-\mu \text { rotrot } \overrightarrow{\mathrm{u}}+\mathrm{Q} \overrightarrow{\mathrm{f}}=0 \tag{1}
\end{equation*}
$$

where $\lambda$ and $\mu$ - Lame coefficients determined by the formulas

$$
\lambda=\frac{v \mathrm{E}}{(1-2 v)(1+v)}, \boldsymbol{\mu}=\frac{\mathrm{E}}{2(1+v)} ; \vec{u} \text { - displacement vector, } Q \vec{f}-\text { vector of mass }
$$

forces. Operators included in the equation [1], to the right of curvilinear orthogonal coordinate systems are defined as follows:

$$
\begin{aligned}
& \operatorname{grad} \phi=\frac{1}{\sqrt{\mathbf{q}_{11}}} \frac{\partial \phi}{\partial \alpha_{1}} \vec{i}_{1}+\frac{1}{\sqrt{\mathbf{q}_{22}}} \frac{\partial \phi}{\partial \alpha_{2}} \vec{i}_{2}+\frac{1}{\sqrt{\mathbf{q}_{33}}} \frac{\partial \phi}{\partial \alpha_{3}} \vec{i}_{3}, \operatorname{rotu}=\frac{1}{\sqrt{\mathbf{q}}} \mathbf{G} \\
& \operatorname{diva}=\frac{1}{\sqrt{\mathbf{q}}}\left[\frac{\partial}{\partial \alpha_{1}}\left(\overrightarrow{\mathbf{u}}_{1} \sqrt{\frac{\mathbf{q}}{\mathbf{q}_{11}}}\right)+\frac{\partial}{\partial \alpha_{2}}\left(\vec{u}_{2} \sqrt{\frac{\mathbf{q}}{\mathbf{q}_{22}}}\right)+\frac{\partial}{\partial{\alpha_{3}}^{2}}\left(\vec{u}_{3} \sqrt{\frac{\mathbf{q}}{\mathbf{q}_{33}}}\right)\right] \\
& \mathbf{G}=\left\lvert\, \begin{array}{lll}
\sqrt{\mathbf{q}_{11}} \overrightarrow{\mathbf{i}}_{1} & \sqrt{\mathbf{q}_{22} \overrightarrow{\mathbf{i}}_{2}} & \sqrt{\mathbf{q}_{33}} \overrightarrow{\mathbf{i}}_{3} \\
\frac{\partial}{\partial \boldsymbol{\alpha}_{1}} & \frac{\partial}{\partial \boldsymbol{\alpha}_{2}} & \frac{\partial}{\partial \boldsymbol{\alpha}_{3}} \\
\mathbf{u}_{1} \sqrt{\mathbf{q}_{11}} & \mathbf{u}_{2} \sqrt{\mathbf{q}_{22}} & \mathbf{u}_{3} \sqrt{\mathbf{q}_{33}}
\end{array}\right.
\end{aligned}
$$

where $\alpha_{i^{-}}$curvilinear coordinates ( $\mathrm{i}-1,3$ ), $\mathrm{q}_{\mathrm{ij}}$ - components of the metric tensor, defined by the equation: $\mathrm{q}_{\mathrm{ij}}=\sum_{\mathrm{k}=1}^{3} \frac{\partial \mathrm{x}_{\mathrm{k}}}{\partial \alpha_{\mathrm{i}}} \frac{\partial \mathrm{x}_{\mathrm{k}}}{\partial \alpha_{\mathrm{j}}} \quad, \mathrm{x}_{\mathrm{k}}$-Cartesian coordinates $(\mathrm{k}=1,3)$, q -squared transformation Jacobean coordinates and Cartesian curvilinear coordinate system. At the same time for orthogonal curvilinear
coordinates only the diagonal terms of the tensor matrix $\mathrm{q}_{\mathrm{ij}}$ not equal to zero. In this case $\mathrm{q}=\sqrt{\prod_{\mathrm{i}=1}^{3} \mathrm{q}_{\mathrm{ii}}}$, and the main differential quadratic form is determined by the equation: $\mathrm{ds}^{2}=\sum_{\mathrm{i}=1}^{3} \mathrm{q}_{\mathrm{ii}} \mathrm{d}^{2} \alpha_{1}$. To determine the stress state of the soil and setting mixed boundary conditions it is necessary to have a formula expressing the voltage across the movement. Using geometric equations derived Novitsky V:

$$
\begin{align*}
& \varepsilon_{i i}=\frac{\partial}{\partial \alpha_{i}}\left(\frac{u_{i}}{h_{i}}\right)+\frac{1}{2 h_{i}^{2}} \sum_{j=1}^{3} \frac{\partial \tilde{h}_{i}^{2}}{\partial \alpha_{j}} \frac{u_{j}}{h_{j}}  \tag{2}\\
& \varepsilon_{\mathrm{ij}}=\frac{1}{2 \mathbf{h}_{\mathrm{i}} \mathbf{h}_{\mathrm{j}}}\left[\mathbf{h}_{\mathrm{i}}^{2} \frac{\partial}{\partial \alpha_{\mathrm{i}}}\left(\frac{\mathbf{u}_{\mathrm{i}}}{\mathbf{h}_{\mathrm{i}}}\right)+\mathbf{h}_{\mathrm{j}}^{2} \frac{\partial}{\partial \alpha_{\mathrm{i}}}\left(\frac{\mathbf{u}_{\mathrm{j}}}{\mathbf{h}_{\mathrm{j}}}\right)\right] \mathrm{i} \neq \mathrm{j}, \quad \mathrm{j}=\overline{1,3}
\end{align*}
$$

In addition, we use the equation of state (Hooke's law) [2]

$$
\begin{equation*}
\sigma_{\mathrm{ij}}=\lambda \delta_{\mathrm{ij}} \sum_{\mathrm{k}=1}^{3} \varepsilon_{\mathrm{kk}}+2 \mu \varepsilon_{\mathrm{ij}} \tag{3}
\end{equation*}
$$

Substituting (2) into (3) we obtain

$$
\begin{equation*}
\sigma_{i j}=\frac{\mu}{h_{i} h_{j}}\left[h_{i}^{2} \frac{\partial}{\partial \alpha_{i}} \frac{u_{i}}{h_{i}}+h_{j}^{2} \frac{\partial}{\partial \alpha_{i}} \frac{u_{j}}{h_{j}}\right], \quad i \neq j \tag{4,б}
\end{equation*}
$$

where $h_{i}^{2}=q_{i i}$. Now we pose the problem of linear elasticity theory for settlement schemes in cylindrical coordinates $\mathrm{r}, \theta$ and z . As the use of unknown components of the displacement vector $u_{r}, u_{\theta}$ и $u_{z}$. The cylindrical coordinate system is associated with a Cartesian coordinate system, the following relationships:

$$
\begin{equation*}
\mathrm{x}=\mathrm{r} \cos \theta ; \quad \mathrm{y}=\mathrm{r} \sin \theta, \quad \mathrm{z}=\mathrm{z}, \mathrm{ds}^{2}=\mathrm{dr}^{2}+\mathrm{r}^{2} \mathrm{~d} \theta^{2}+\mathrm{dz}^{2} \tag{5}
\end{equation*}
$$

Using equations (5), we obtain

$$
\mathbf{h}_{1}^{2}=\mathbf{h}_{3}^{2}=\mathbf{q}_{11}=\mathbf{q}_{33}=1, \quad \mathbf{h}_{22}^{2}=\mathbf{q}_{22}=\mathbf{r}^{2}
$$

As an coordinates $\alpha_{i}(i=1,3)$ applicable:

$$
\begin{equation*}
\alpha_{1}=\mathrm{r}, \alpha_{2}=\theta, \alpha_{3}=\mathrm{z} \tag{6}
\end{equation*}
$$

Substituting (5) and (6) (1), and the resulting expression into the formula (4) and taking into account the following system of Lame equations in cylindrical coordinates:

$$
\begin{aligned}
&(\lambda+2 \mu)\left(u_{r}\right)_{\mathrm{rr}}+\frac{\mu_{2}}{\mathrm{r}}\left(\mathrm{u}_{\mathrm{r}}\right)_{\theta \theta}+\mu\left(\mathrm{u}_{2}\right)_{\mathrm{zz}}+\frac{\lambda+\mu}{\mathrm{r}}\left(\mathrm{u}_{\theta}\right)_{\theta \mathrm{r}}+(\lambda+\mu)\left(\mathrm{u}_{\mathrm{z}}\right)_{\mathrm{zz}}+ \\
&+\frac{\lambda+2 \mu}{\mathrm{r}}\left(\mathrm{u}_{\mathrm{r}}\right)_{\mathrm{r}}-\frac{\lambda+3 \mu}{\mathrm{r}^{2}}\left(\mathrm{u}_{\theta}\right)_{\theta}-\frac{\lambda+2 \mu}{\mathrm{r}^{\mathrm{r}}} \mathrm{u}_{\mathrm{r}}=0 \\
& \mu\left(\mathrm{u}_{\theta}\right)_{22}+ \frac{\lambda+2 \mu}{\mathrm{r}^{2}}\left(\mathrm{u}_{\theta}\right)_{\theta}+\mu\left(\mathrm{u}_{\theta}\right)_{\mathrm{zz}}+\frac{\lambda+\mu}{\mathrm{r}}\left(\mathrm{u}_{\mathrm{r}}\right)_{\mathrm{r} \theta}+\frac{\lambda+\mu}{\mathrm{r}}\left(\mathrm{u}_{\mathrm{z}}\right)_{\mathrm{z} \theta}+ \\
&+\frac{\mu}{\mathrm{r}^{2}}\left(\mathrm{u}_{\mathrm{r}}\right)_{\theta}-\frac{\mu}{\mathrm{r}} \mathbf{u}_{\theta}=0, \\
& \mu\left(\mathrm{u}_{\mathrm{z}}\right)_{\mathrm{rr}}+ \frac{\mu}{\mathrm{r}^{2}}\left(\mathrm{u}_{\theta}\right)_{\theta \theta}+(\lambda+2 \mu)\left(\mathrm{u}_{\mathrm{z}}\right)_{\mathrm{zz}}+(\lambda+\mu)\left(\mathrm{u}_{\mathrm{r}}\right)_{\mathrm{rr}}+ \\
&+\frac{\lambda+\mu}{\mathrm{r}}\left(\mathrm{u}_{\theta}\right)_{\theta \mathrm{z}}+\frac{\lambda+\mu}{\mathrm{r}}\left(\mathrm{u}_{\mathrm{r}}\right)_{\mathrm{z}}=0
\end{aligned}
$$

where the indices $\mathrm{r}, \theta$ and z , outside the brackets denote partial derivatives with respect to the corresponding coordinates. The boundary conditions on the external surface of the tube - a condition of perfect contact with the ground, the inner surface is free:

$$
\begin{align*}
& r=R: u_{r 1}=u_{r 2}, u_{\theta 1}=u_{\theta 2}, u_{z}=u_{z 2}, \\
& \sigma_{r r 1}=\sigma_{r r 2}, \sigma_{r \theta 1}=\sigma_{r \theta 2}, \sigma_{r z 1}=\sigma_{r 22},  \tag{8}\\
& r=R_{0}: \sigma_{r r 2}=0, \sigma_{r \theta 1}=0, \sigma_{r z 1}=0,
\end{align*}
$$

Where the subscripts " 1 " and " 2 " respectively denote materials ambient environment and the tube. The boundary conditions to ensure equality of the normal components of the fluid velocity and the shell are

$$
\left.\left(\begin{array}{ll}
\vec{V} & \overrightarrow{\mathbf{n}} \tag{9}
\end{array}\right)\right|_{\left\lvert\,=+\frac{\hat{\mathbf{u}}_{\mathrm{r} 2}}{\vec{a}}\right.} ^{r=a}
$$

where $\vec{v}$ - the fluid velocity of the particle; $\mathbf{n}$ - the surface normal atr $=a, w$ - radial movement of the shell. To fully close the formulation of the problem, it is necessary to the conditions (8) and (9) to add conditions at infinity $\mathbf{u} \longrightarrow \mathbf{0}$
at $\mathrm{R}=\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}} \rightarrow \infty \quad$,
filled with some conditions on radiation.
For non-stationary problems as radiation conditions required to fulfill the principle of causality, and environment should be no movement outside the region bounded by the leading edge of the waves from oscillation sources.


Fig. 1. Calculated scheme
Consider the problem of dynamic linear elasticity theory on the effects of seismic waves in the pipes, laid in a high mound in two lines and filled with ideal compressible fluid. In this case, we consider the case when the wave is incident perpendicular to the axis connecting the centers of the pipes, and to the longitudinal axis of the pipe. Design scheme is presented in Figure 1. Bi cylindrical coordinate system associated with a Cartesian coordinate system, the following relationships:

$$
\begin{equation*}
\mathrm{x}=(\operatorname{asin} \xi) /(\operatorname{ch} \eta-\cos \xi), \quad \mathrm{y}=(\operatorname{ash} \eta) /(\operatorname{ch} \eta \cos \xi), \quad \mathrm{z}=\mathrm{z} \tag{11}
\end{equation*}
$$

where: a - half the distance between points $\eta=-\infty$ and $\eta=\infty$.
Then, introducing (11) to (5.6), and the resulting expression in (6) take the following form:

$$
\begin{equation*}
\mathrm{ds}^{2}=\mathrm{a}^{2}(\operatorname{ch} \eta-\cos \xi)^{-2} \mathrm{~d} \xi^{2}+\mathrm{a}^{2}(\operatorname{ch} \eta-\cos \xi)^{-2} \mathrm{~d} \eta^{2}+\mathrm{dz}^{2} \tag{12}
\end{equation*}
$$

Using equations (11), we obtain

$$
\begin{equation*}
\mathbf{h}_{1}^{2}=\mathbf{h}_{2}^{2}=\mathbf{q}_{11}=\mathbf{q}_{22}=\mathbf{a}^{2}(\operatorname{ch} \eta-\cos \xi)^{-2}, \mathrm{~h}_{3}^{2}=\mathrm{q}_{33}=1 \tag{13}
\end{equation*}
$$

Assuming that: $\alpha_{1}=\xi, \alpha_{2}=\eta, \alpha_{3}=z$ and substituting (12) and (13) in (1) - (11), and, given that the task is flat, we obtain the following equation in bipolar coordinates Helmholtz:

$$
\begin{align*}
& {\left[a^{-2}(\operatorname{ch\eta }-\cos \xi)^{2}\right]\left[(v)_{\xi \xi}+(v)_{\eta \eta}\right]+k^{2} v=0}  \tag{14}\\
& \text { where } \frac{\sin \xi}{\operatorname{chn}-\cos \xi}= \begin{cases}2 \sum_{n=1}^{\infty} e^{-n \eta} \sin n \xi & \boldsymbol{\eta}>0 \\
2 \sum_{n=1}^{\infty} e^{n \eta} \sin n \xi & \eta<0\end{cases}
\end{align*}
$$

Equation (14) after several transformations is reduced to the form

$$
\begin{equation*}
(\mathrm{v})_{\xi \xi}+(\mathrm{v})_{\eta \eta}+\left(2 \mathrm{kae}^{ \pm \eta}\right)^{2} \mathrm{v}=0 \tag{16}
\end{equation*}
$$

Solution of the equation (14) will be sought in the form of a series:

$$
\begin{equation*}
v=\sum_{n=0}^{\infty}\left[v_{n}^{a}(\eta) \cos n \xi+v_{n}^{b}(\eta) \sin n \xi\right] \varepsilon^{-i w t} \tag{17}
\end{equation*}
$$

Substituting (17) into (16) and equating the coefficients of the respective harmonics, we obtain the following ordinary differential equation:

$$
\begin{equation*}
\mathrm{v}_{\mathrm{n}}^{\prime \prime}+\left[\left(2 \mathrm{kae}^{ \pm \eta}\right)^{2}-\mathrm{n}^{2}\right] \mathrm{v}_{\mathrm{n}}=0 \tag{18}
\end{equation*}
$$

Standard replacement

$$
\mathbf{V}_{\mathbf{n}}(\eta)=\mathbf{z}(\mathbf{t}) \quad, \mathrm{t}=\exp ( \pm \eta)
$$

reduce (18) to the Bessel equation of the form

$$
\begin{equation*}
t^{2} z^{\prime \prime}+t z^{\prime}+\left(4 k^{2} a^{2}-n^{2}\right) z=0 \tag{19}
\end{equation*}
$$

which has a particular solution in the form of cylindrical functions $z\left(2 \mathrm{ake}^{-\eta}\right)$, and the solution of the Helmholtz equation takes the following form:

$$
\begin{align*}
& \varphi=\sum_{n=0}^{\infty} A_{n} Z_{n}\left(2 a k e^{\mp \eta}\right) \cos n \xi e^{-i w t}, \\
& \psi=\sum_{n=0}^{\infty} B_{n} Z_{n}\left(2 a k e^{\mp \eta}\right) \sin n \xi e^{-i w t} \tag{20}
\end{align*}
$$

Now put the boundary conditions. To do this, use the condition (20), the replacement $\mathrm{r}=\eta$ and $\theta=\xi$. Considering these relations, we seek a solution of the boundary value problem for the case fall into two
underground pipes P - compression waves and shear SV-wave perpendicular to the axis y . Wave potential wave has the form

$$
\begin{equation*}
\varphi^{(i)}=A e^{\mathrm{i} \alpha \cdot \mathrm{x}-\mathrm{iwt}} \tag{21}
\end{equation*}
$$

For the representation (21) as (20), write (21) through (12) in the bipolar cylindrical coordinates.

$$
\begin{equation*}
\varphi_{1}^{(i)}=A e^{i k 2 a \exp (\mp)^{\eta} \sin \xi e^{-i w t}} \tag{22}
\end{equation*}
$$

Expanding the second factor of (22) in a Fourier series (integrated form), and after some transformations we obtain the final expression for the potential of the incident P - waves:

$$
\begin{equation*}
\varphi_{1}^{(i)}=A \sum_{n=0}^{\infty} \varepsilon_{n} J_{n}\left(\alpha_{1} \tau\right) \cos n \xi e^{-i w t} \tag{23}
\end{equation*}
$$

where $\tau=2 \operatorname{aexp}(\bar{\mp})$
Other potential (20) analogous to (23) have the form:

$$
\begin{gather*}
\varphi_{2}^{(r)}=\sum_{n=0}^{\infty}\left[C_{n} H_{n}^{(1)}\left(\alpha_{2} \tau\right)+D_{n} H_{n}^{(2)}\left(\alpha_{2} \tau\right) \cos n \xi e^{-i w t}, \psi_{2}^{(r)}=\sum_{n=0}^{\infty}\left[E_{n} H_{n}^{(1)}\left(\beta_{2} \tau\right)+F_{n} H_{n}^{(2)}\left(\beta_{2} \tau\right) \sin n \xi e^{-i w t}\right.\right.  \tag{24}\\
\varphi_{3}^{(r)}=\sum_{n=0}^{\infty} G_{n} J_{n}^{(1)}\left(\alpha_{3} \tau\right) \cos n \xi e^{-i w t}
\end{gather*}
$$

Dynamic VAT is expressed in terms of the potentials $\varphi_{1}$ and $\psi_{2}$ :

$$
\begin{align*}
& u_{\eta i}=\delta\left[\left(\varphi_{i}\right)_{\eta}-\left(\psi_{i}\right)_{\xi}\right] u_{\xi i}=\delta\left[\left(\varphi_{i}\right)_{\xi}-\left(\psi_{i}\right)_{\eta}\right], u_{\eta 3}=-\delta(i w)^{-1}\left(\varphi_{3}\right)(25)  \tag{25}\\
& \sigma_{\eta \eta i}=-\sigma_{\xi \xi i}=2 \delta^{2}\left\{d_{i}\left[0,5 \varphi_{\eta \eta}-\left(\varphi_{\xi}+\varphi_{\eta}\right) \sin \xi\right]+0,5 \lambda_{i} \varphi_{\xi \xi}-\mu_{i}\left(\psi_{\xi \xi}-\varphi_{\eta}+\psi_{\xi}\right)\right\} \\
& \tau_{\eta \eta 3}=\sigma_{\xi \xi 3}=-i w_{3} \rho_{3} \varphi_{3}, \tau_{\eta \xi i}=2 \mu_{i} \delta^{2}\left[\varphi_{\xi \eta}+0,5 \psi_{\eta \eta}-0,5 \psi_{\xi \xi}+\varphi_{\xi}+\psi_{\eta}+\left(\varphi_{\xi}-\psi_{\xi}\right) \sin \xi\right] \\
& \mathbf{i}=\mathbf{1}, 2 ; \delta=\mathbf{e}^{\mp \eta} / 2 \mathbf{a} .
\end{align*}
$$

Substituting (24) and (25) (8), we obtain the final solution of problems of the fall, respectively P and SV - waves on two underground pipes. The arbitrary constants $A_{n}, B_{n}, C_{n}$ andet al. are determined from the system of algebraic equations with complex coefficients:

$$
[C]\{q\}=\{\rho\}
$$

whereC - determinant ( $12 \times 12$ ) - the order of the elements of which are a function of Bessel and Henkel 1st 2nd kind of n-th order of $q$ - column vector of unknowns, $\rho$ - the vector of the right.

Gauss solve the system of algebraic equations with complex coefficients with the release of the main element. Dynamic VAT in the event of a fall - the shear waves in two underground pipes recorded in bipolar coordinates in the asymptotic form:

$$
\mathrm{u}_{\mathrm{z}}=\mathrm{w}, \sigma_{\eta \mathrm{z}}=\mu_{\mathrm{i}} \delta\left(\mathrm{u}_{\mathrm{z}}\right)_{\eta}, \sigma_{\xi \mathrm{z}}=\mu_{\mathrm{i}} \delta\left(\mathrm{u}_{\mathrm{z}}\right)_{\xi}
$$

As an condition of use of the boundary conditions (23) and substitute $r=n$. The final solution of the problem for cases falling SH - wave on the two tubes has the form:

$$
\begin{align*}
& u_{z 1}=w_{0} \sum_{n=0}^{\infty}\left[\varepsilon_{n} J_{n}\left(k_{1} \tau\right)+A_{n} H_{n}^{(1)}\left(k_{1} \tau\right)\right] \cos n \xi e^{-i w t} ; u_{z 2}=-w_{0} \sum_{n=0}^{\infty}\left[B_{n} H_{n}^{(1)}\left(k_{2} \tau\right)+C_{n} H_{n}^{(2)}\left(k_{2} \tau\right)\right] \cos n \xi e^{-i w t} ; \\
& \sigma_{r 21}=\mu_{1} w_{0} k_{1} \sum_{n=0}^{\infty}\left[\varepsilon_{n} J_{n}\left(k_{1} \tau\right)+A_{n} H_{n}^{(1)}\left(k_{1} \tau\right)\right] \cos n \xi e^{-i w t} ; \sigma_{r 22}=-\mu_{2} w_{0} k_{2} \sum_{n=0}^{\infty}\left[B_{n} H_{n}^{(1)}\left(k_{2} \tau\right)+C_{n} H_{n}^{(2)}\left(k_{2} \tau\right)\right] \cos n \xi e^{-i w t} ;  \tag{27}\\
& \sigma_{\theta: 1}=-\mu_{1} w_{0} n \sum_{n=0}^{\infty}\left[\varepsilon_{n}\left(k_{1} \tau\right)+A_{n} H_{n}^{(1)}\left(k_{1} \tau\right)\right] \sin n \xi e^{-i w t} ; \sigma_{\theta: 2}=\mu_{2} w_{0} n \sum_{n=0}^{\infty}\left[B_{n} H_{n}^{(1)}\left(k_{2} \tau\right)+C_{n} H_{n}^{(2)}\left(k_{2} \tau\right)\right] \sin n \xi e^{-i w t} ;
\end{align*}
$$

Uncertain factors $A_{n}, B_{n}, C_{n}$ determined from the boundary conditions.
Consider the definition of dynamic stress-strain state of a cylindrical tube under the influence of harmonic waves.

To solve this problem use the addition theorem. The addition theorem for cylindrical wave functions derived in $[4,5,6]$. Suppose there are two different polar coordinate system $\left(\mathrm{r}_{\mathrm{g}}, \theta_{\mathrm{g}}\right)$ and $\left(\mathrm{r}_{\mathrm{k}}, \theta_{\mathrm{k}}\right)$ (Figure 3), in which the polar axis of the same direction. pole coordinate $\theta_{\mathrm{k}} \mathrm{q}$ in the system will be $\mathrm{R}_{\mathrm{kq}}$, $\theta_{\mathrm{kq}}$, so that the equality

$$
\begin{equation*}
Z_{g}=R_{k g} e^{i \theta_{k g}}+Z_{k} \tag{28}
\end{equation*}
$$

Then the addition theorem has the form:

$$
\begin{align*}
& b_{n}\left(\alpha r_{q}\right) e^{i n \theta_{q}}=\sum_{p=-\infty}^{\infty} b_{n-p}\left(\alpha R_{k q}\right) e^{i(n-p) \theta_{k q}} T p\left(\alpha r_{k}\right) \exp \left(i p \theta_{k}\right), r_{k}<R_{k q}, \\
& b_{n}\left(a r_{q}\right) e^{i n \theta_{q}}=\sum_{p=-\infty}^{\infty} J_{n-p}\left(\alpha R_{k q}\right) e^{i(n-p) \theta_{k q}} b_{p}\left(\alpha r_{k}\right) \exp \left(i p \theta_{k}\right), r_{k}<R_{k q} \tag{29}
\end{align*}
$$

Equation (28) makes it possible to convert the solution of the wave equation (1) from one coordinate system to another. Consider the calculation of the extended multi-line underground pipeline to the seismic action in the framework of a dynamic theory of elasticity plane problem. In this study the case of stationary diffraction of plane waves at a number of periodically arranged cavities, reinforced rings with ideal compressible fluid inside. The solution of the problem of implementing the method of potentials. The boundary conditions have the form (8). Do not change the form and potential of the incident. The potentials of the reflected waves from the pipe after applying the addition theorem, and taking into account the frequency of the problem, will have the form:

$$
\begin{align*}
& \varphi_{1}^{(r)}=e^{-i w t} \sum_{n=0}^{\infty}\left[A_{n} H_{n}^{(1)}\left(\alpha_{1} r\right)+S_{n} J_{n}\left(\alpha_{1} r\right)\right] e^{i n(\theta-\gamma)}, \\
& \psi_{1}^{(r)}=e^{-i w t} \sum_{n=0}^{\infty}\left[B_{n} H_{n}^{(1)}\left(\beta_{1} r\right)+\sigma_{n} J_{n}\left(\beta_{1} r\right)\right] e^{i n(\theta-\gamma)}, \\
& S_{n}=\sum_{p=0}^{\infty} \sum_{m=1}^{\infty} A_{p} E_{p}\left[e^{i m \xi} H_{n-p}^{(1)}\left(\alpha_{1} m \delta\right)+e^{-i m \xi} H_{n-p}^{(1)}\left(\alpha_{1} m \delta\right)\right],  \tag{30}\\
& Q_{n}=\sum_{p=0}^{\infty} \sum_{m=1}^{\infty} B_{p} E_{p}\left[e^{i m \xi} H_{n-p}^{(1)}\left(\beta_{1} m \delta\right)+e^{-i m \xi} H_{n-p}^{(1)}\left(\beta_{1} m \delta\right)\right],
\end{align*}
$$

where: $\xi=\mathrm{k} \delta \cos \gamma, \delta$ - distance between pipe centers.
Potentials of refracted waves in the pipes can be written as

$$
\begin{align*}
& \varphi_{2}=e^{i(m \xi-w \xi)} \sum_{n=0}^{\infty} E_{n}\left[C_{n} H_{n}^{(1)}\left(\alpha_{1} r\right)+D_{n} H_{n}^{(2)}\left(\alpha_{2} r\right)\right] e^{\operatorname{in}(\theta-\gamma)} \\
& \psi_{2}=e^{i(m \xi-w \xi)} \sum_{n=0}^{\infty} E_{n}\left[E_{n} H_{n}^{(1)}\left(\beta_{1} r\right)+F_{n} H_{n}^{(2)}\left(\beta_{2} r\right)\right] e^{i n(\theta-\gamma)} \tag{31}
\end{align*}
$$

and the velocity potential in the perfect shape of a compressible fluid

$$
\begin{equation*}
\varphi_{3}=e^{i(m \xi-w \xi)} \sum_{n=0}^{\infty} E_{n} G_{n} J_{n}\left(\alpha_{3} r\right) e^{i n(\theta-\gamma)} \tag{32}
\end{equation*}
$$

The unknown coefficients $\mathrm{A}_{\mathrm{n}}$ - $\mathrm{G}_{\mathrm{n}}$ determined by formulation (29) - (32) (8). The result is an infinite system of linear equations, which is solved by an approximate method of reduction, provided that it is not the relation

$$
\mathrm{k} \delta(1 \mp \cos \gamma)=2 \pi \mathrm{n}
$$

General characteristics of the program is designed for multi-line pipes in the case of the mound to the fall of seismic waves perpendicular to the axis passing through the center of the pipe.


Fig. 2. Driving to the addition theorem.
The input information includes the minimum required data: elastic characteristics (Eandv) soil embankments and pipes; the density of the soil, pipe and fluid fills it; the inner and outer radii of the pipes; the predominant period of ground oscillations of the particles; coordinates of the point in which the VAT; seismicity rate. With the help of special tags can count tubes filled with ideal compressible liquid and empty. Calculation of cylindrical Bessel and Hankel functions performed by the known formulas. Solving systems of linear equations Gauss carried out by a member of the main allocation.

Influence of the distance between the pipes. Table 1 shows the values of the coefficient

$$
\eta_{\max }\left(\eta_{\max }=\left|\sigma_{\mathrm{rr}}\right| /(\lambda+2 \mu) \alpha^{2} \mathrm{~A}\right.
$$

maximum radial pressure on the soil pipe at varying distances d between them in the event of a fall F - wave. It was assumed the wave number of P - waves $\alpha_{\mathrm{r}}=1,0$ : the inner and outer radius of the pipe $\mathrm{R}_{0}=0,8 \mathrm{~m}$ and $\mathrm{R}=1,0 \mathrm{~m}$ : prevailing between the soil particles oscillations $\mathrm{T}=0.2 \mathrm{sec}$. Features mound of soil: permanent

Lame $\lambda_{1}=8,9-\mathrm{MPa} ; \quad \mu_{1}=4,34 \mathrm{MPa} ; \quad$ density $\rho_{1}=1,74$ Кнseк $^{2} / \mathrm{m}^{4}$. The characteristics of the pipe material $\lambda_{2}=8690 \mathrm{MPa} ; \mu_{2}=12930 \mathrm{MPa} ; \rho_{2}=2,55$ Кнseк ${ }^{2} / \mathrm{m}^{4}$.


Figure 3. Estimated scheme.
Table 1.The value of dynamic concentration factor for different distances between the pipes for the case of the fall of $P$ - waves

| $\mathrm{D} / \mathrm{d}$ | 0,5 | 1,0 | 2,0 | 4,0 |
| :---: | :---: | :---: | :---: | :---: |
| $\eta_{\max }$ | 1,68 | 1,76 | 1,61 | 1,60 |

It follows from Table 1 that with increasing first distance between tubes $0,5 \leq \mathrm{d} / \mathrm{D} \leq 1,0$ coefficient $\eta_{\max } s$ lightly increased by $5 \%$, with a further increase $d / D>1,0$ decreases more sharply by $10 \%$. At $d / D>2,0$ value $\eta_{\max } s t a b i l i z e d$, i.e. virtually unchanged, with $1 \leq 4,0$ close to the value $\eta_{\max }$ for single pipe according to the calculations. Consequently, the mutual influence of concrete pipes multiline stacking occurs when the distance between them $\mathrm{d} \leq 4,0 \mathrm{D}$ and increases the maximum dynamic soil pressure on them compared with a single tube. This magnification effect $\eta_{\max }$ associated with the combination of waves reflected more surfaces of a multi-tube. This non-monotonic increase coefficient $\eta_{\max }$ with decreasing distance between the pipes $\mathrm{d} / \mathrm{D}$ is connected in our opinion to the phenomenon of interference imposed upon reflection waves. This phenomenon is extremely important for the practice of seismic design of underground pipelines multiline, because It allows you to choose the optimal distance between the tubes, in which the dynamic pressure under seismic impact is minimal. For example, Table 1 in this distance is $d=$ $0,5 \mathrm{D}$. It is known, be noted for comparison that in the case of the static effects the opposite is true: the ground pressure of a multi-tube smaller than a single. In addition to the above, in the analysis of the distance between the pipes influencing their VAT must consider the relation (28) (so-called "slip point") at which there is a significant increase in the dynamic stresses in the vicinity of the tube - resonance. This phenomenon is known under the name of Wood's optical anomaly is a feature of a multi-line and can not arise in the pipeline, stacked in a single thread. In terms of design practice, it is necessary to know how far you can lay the pipe to the dangerous phenomenon of resonance does not occur. The answer to this question is given by equation (27). Let us analyze this relationship in the case of the impact of P - and SV - seismic waves on the underground pipeline. Table 2 shows the dependence of the maximum distance between the centers of the light pipe $d_{\text {max }}$, wherein no resonance occurs, the angle of incidence of the seismic waves $\gamma$.

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table 2. The dependence of the distance $D_{\text {max }}$ the angle of incidence $\gamma$.

| $\gamma$. Град | 0 | 30 | 45 | 60 | 70 | 80 | 90 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| $\mathrm{D}_{\text {max }}, \mathrm{M}$ | 5,0 | 5,36 | 5,86 | 6,66 | 7,45 | 8,52 | 10,0 |

From Table 2, the smaller the angle of incidence of the seismic wave on the pipe, the closer to each other is necessary to lay the pipe. Thus, the appearance of a multi-resonance tubes can be avoided by selecting an appropriate distance between them and thereby provide earthquake resistance conduit. Influence of the type of the seismic action ( $\mathrm{P}, \mathrm{SV}$-or SH -wave). Table 3 shows the values $\eta_{\text {max }} s o i l$ maximum radial pressure on the pipe when $P$ - and downs SV - seismic waves at varying distances $d$ between the pipes. It we assumed $\beta_{\mathrm{r}}=2$. Analysis of the data table. 3 shows that when $\mathrm{d} / \mathrm{D}<4,0$ coefficient values $\eta_{\text {max }}$ for P -wave and SV -like are in antiphase, i.e. in $1 / \mathrm{D}=1,0$ seismic action maximum P-wave is $27 \%$ higher than that of SV - wave when $\mathrm{d} /$ $D=2,0 \%$ below 7 , and with $d / D=4,0$ up again, but only $1 \%$.If this distance increases with the difference between the tubes in these effects is reduced and $d / D=4,0$ disappears almost altogether. In addition, we note that when exposed to $S V$ - wave values $\eta_{\max }$ at various distances between the tubes is 2.5 times the variation $(25 \%)$ than when exposed F - wave ( $10 \%$ ). Thus, the phenomenon of "local resonance" appears more strongly to the seismic action as SV- wave.

Table 3. The value of the coefficient $\eta_{\text {max }}$ under seismic actions as F - and SV - waves at various distances $d$ between the tubes

| $\mathrm{d} / \mathrm{D}$ | $\eta_{\max }$ |  |
| :---: | :---: | :---: |
|  | P - волна |  |
| 1,0 | 1,76 | SV - wave |
| 2,0 | 1,61 | 1,29 |
| 4,0 | 1,60 | 1,72 |

Influence of liquid filling tube.Table 4 shows the values of the coefficient $\eta_{\max }$ in the case of the fall of P - waves on empty and water-filled tube at different distances d between the pipes. Liquid density was assumed to be $\rho_{3}=0,102$ Кнзек ${ }^{2} / \mathrm{m}^{4}$.

Table 4. The value of the coefficient $\eta_{\text {max }}$ in the case of the fall of $P$ - waves on empty and water-filled tube

| $\mathrm{d} / \mathrm{D}$ | $\eta_{\max }$ |  |
| :---: | :---: | :---: |
|  | P - wave | SV - wave |
| 1,0 | 1,76 | 1,89 |
| 2,0 | 1,61 | 1,78 |
| 4,0 | 1,60 | 1,90 |

It follows from the Table 4 that the presence of water in the pipes increases seismic influence on them compared to empty tubes. Obviously, this is associated with increased weight of the pipeline. The maximum dynamic pressure on the soil pipe is enhanced. For example, when $d / D=1,0$ coefficient difference values $d$ $/ \mathrm{D}=2,0-10 \%$, with $\mathrm{d} / \mathrm{D}=4,0-19 \%$.

In addition, we note that the coefficient of variation values $\eta_{\max }$ at various distances $d$ tubes filled with less water ( $7 \%$ ) than in the empty pipes ( $10 \%$ ).

Effect of the incident seismic wave length.Table 4 shows the values of the coefficient $\eta_{\max }$ different lengths $1_{0} / l_{0}-2 \pi / \alpha, \mathrm{p}$ - wave incident on the empty pipe, located at a distance $\mathrm{l}=1,0 \mathrm{D}$ from each other.

Table 5.The values of the coefficient $\eta_{\text {max }}$ for different lengths $l_{0} P$ - waves.

| $\mathrm{l}_{0} / \mathrm{D}$ | 3,0 | 5,0 | 10,0 |
| :---: | :---: | :---: | :---: |
| $\eta_{\max }$ | 1,76 | 1,52 | 1,20 |

From Table. 5 that the greater the length of the incident seismic wave; denser than the soil mound, the lower the ratio $\eta_{\text {max }}$. For the record, we note that the ratio $\mathrm{l}_{0} / \mathrm{D}=5,0$ - not bulk sand, sandy loam and loamy soils; $l_{0} / \mathrm{D}=10,0$ - clayey soils. Thus, the type of soil, and in particular its density has a significant impact on its dynamic pressure in the pipe when the seismic action. It follows that in the construction of the embankment above the pipe must be carefully compacted backfill. Interestingly, good compaction and reduces the static pressure on the pipe. Moreover, calculations show that for $1_{0}>10,0 \mathrm{D}$ dynamic problem reduces to the quasistatic, which greatly simplifies the solution. Hence an important conclusion that the quasi-static approach can not be applied to the calculation of seismic impact on culverts.

Effect of the pipe wall thickness.Table 6 shows the values of the coefficient $\eta_{\text {max }}$ tolschinyt for various wall concrete pipe in the event of a fall of P - waves on empty multiline pipes stacked multi-line pipe, laid on the distance $\mathrm{d}=0,5$.

Table 6. The value of the coefficient $\eta_{\text {max }}$ for different pipe wall thickness $t$

| $\mathrm{d} / \mathrm{D}$ | 0,08 | 0,1 | 0,15 | 0,2 |
| :---: | :---: | :---: | :---: | :---: |
| $\eta_{\max }$ | 1,60 | 1,66 | 1,66 | 1,68 |

It follows from the Table 6 that the range of the wall thickness, virtually no effect on the dynamic pressure does not soil the pipes. This is apparently due to the fact that the harmonic wave does not pass into the concrete pipe sufficient stiffness to force the tube.

## Conclusions.

1. When the seismic action the mutual influence of concrete pipes of a multi-stacking occurs when the distance between them $\mathrm{d}>4,0 \mathrm{D}$ and increases the maximum dynamic pressure of soil on them compared with a single pipe (local resonance phenomenon) by $5-10 \%$.
2. The emergence of multi-resonance pipes can be avoided if you choose the distance between the nonmultiple lengths of the incident seismic wave. This resonance phenomenon is a characteristic of a multi-line and cannot occur in a pipe string stacked one.
3.local resonance phenomenon manifests itself more strongly to the seismic action in the form of SV - waves than P - waves.
3. The presence of water in the pipes increases seismic influence on them by $10-20 \%$.
4. The denser the soil mound, the lower the seismic impact on the underground pipes. at $1>10 \mathrm{D}$ dynamic problem reduces to the quasi-static.
5. Change the wall thickness and concrete class has virtually no effect on the dynamic pressure of the ground with reinforced concrete pipe under the seismic action.
similar dependence is also built when $\gamma=0$. It is interesting to note that in this problem, increase the stress concentration due to the proximity of other field gap, much more when the wave is incident from the side (i.e. $\gamma=0$ ), than wave falls from above (i.e. $\gamma=\pi / 2$ ).
6. The maximum dynamic soil pressure $\sigma_{\text {max }}$ pipes, arranged in two lines at a distance $\mathrm{d}<3,0 \mathrm{D}$ from each other by more than a single tube. This excess is $15 \%$.
7. The liquid in the tubes, usually increases the pressure $\sigma_{\max }$ for a single tube by $20 \%$, and for two of thread Pipe $5-10 \%$. The exception is tightly packed tube $\mathrm{d}=0$, for which the pressure $\sigma_{\max }$ decreases by $4 \%$.

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